Answer Key

Aktosun, Spring 2017, Math 3319, Exam 3

Questions 1-15: True/False. Questions 16-30: Multiple-Choice.

1. If A is a 45×45 matrix, then the matrix size of e^{A^2} must also be 45×45. T

2. The zero vector may be an eigenvector for some matrix. F

3. There exists a 7×7 matrix with the scalar 8 as an eigenvalue of multiplicity seven. T

4. The scalar zero can never be an eigenvalue for any matrix. F

5. The general solution to the system $\mathbf{y}' = \mathbf{A}\mathbf{y}$, where $\mathbf{y}$ is a column vector with n components that are functions of x, $\mathbf{A}$ is a constant $n \times n$ matrix, and the prime denotes the derivative with respect to x, is given by $\mathbf{y} = e^{\mathbf{A}x}\mathbf{c}$, where $\mathbf{c}$ is a column vector with n components, each of which is an arbitrary constant. T

6. There exists a matrix A whose eigenvalues are $2+3i$, $-2+3i$, and $3-2i$. T

7. A 1202×1202 matrix may have 1203 distinct eigenvalues. F

8. The determinant of a square matrix is equal to the product of its eigenvalues. T

9. The general solution to the ODE $y'' + 2y' + y = 0$, where the prime denotes the derivative with respect to x, is given by $y = c_1 e^{-x} + c_2 e^{-x}$, where c_1 and c_2 are arbitrary constants. F

10. A square matrix cannot be invertible unless all its eigenvalues are nonzero. T

11. If λ is an eigenvalue of the matrix A, then λ^3 must be an eigenvalue of the matrix A^3. T

12. There exists a 5×5 matrix having only one linearly independent eigenvector. T

13. For a diagonal matrix, each diagonal entry is an eigenvalue. T

14. The trace of a square matrix is equal to the sum of its eigenvalues. T

15. All eigenvalues of a real symmetric matrix must be real. T

16. If $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ and if it is known that $e^{At} = \begin{bmatrix} e^t & e^t - e^{2t} \\ 0 & e^{2t} \end{bmatrix}$, find the general solution to the system $\begin{cases} y_1' = y_1 - y_2 \\ y_2' = 2y_2 \end{cases}$ where t denotes the independent variable.

(a) $y_1 = c_1\, e^t, \quad y_2 = c_2\, e^{2t}$ (b) $y_1 = c_1\, e^t, \quad y_2 = c_1\, e^t + c_2\, e^{2t}$

(c) $y_1 = c_1\, e^t + c_2\, e^{2t}, \quad y_2 = c_2\, e^t + c_1\, e^{2t}$

(d) $y_1 = (c_1 + c_2)\, e^t - c_2\, e^{2t}, \quad y_2 = c_2\, e^{2t}$

17. For any square matrix A, the inverse of the matrix e^A is given by

(a) $e^{A^{-1}}$ (b) $e^{-A^{-1}}$ (c) e^{-A} (d) $e^{(-A)^{-1}}$

18. Find the general solution to the ODE given by $D^3(D-2)(D^2+4)\, y = 0$, where D denotes the differential operator d/dx.

(a) $y = c_1 + c_2 + c_3 + c_4\, e^{2x} + c_5 \cos(2x) + c_6 \sin(2x)$

(b) $y = c_1 + c_2 + c_3 + c_4\, e^{-2x} + c_5 \cos(2x) + c_6 \sin(2x)$

(c) $y = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x} + c_5 \cos(2x) + c_6 \sin(2x)$

(d) $y = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{2x} + c_5 \cos(2x) + c_6 \sin(2x)$

19. Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$.

(a) eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for eigenvalue 1, eigenvector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ for eigenvalue 2

(b) eigenvector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ for eigenvalue 4, eigenvector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ for eigenvalue 0

(c) eigenvector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ for eigenvalue 2, eigenvector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ for eigenvalue 2

(d) eigenvector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for eigenvalue -2, eigenvector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for eigenvalue -2

20. Find the general solution to $y'' - y = e^x + e^{-x}$.

(a) $y = c_1\, e^x + c_2\, e^{-x} + e^x + e^{-x}$ (b) $y = c_1\, e^x + c_2\, e^{-x} + 2\, e^x + 2\, e^{-x}$

(c) $y = c_1\, e^x + c_2\, e^{-x} + \frac{1}{2}\, x\, e^x + \frac{1}{2}\, x\, e^{-x}$ (d) $y = c_1\, e^x + c_2\, e^{-x} + x\, e^x + x\, e^{-x}$

21. Find the general solution to $y'' + 6\, y' + 9\, y = 0$.

(a) $y = c_1\, e^{3x} + c_2\, e^{-3x}$ (b) $y = c_1\, e^{-3x} + c_2\, x\, e^{-3x}$ (c) $y = c_1\, e^{3x} + c_2\, x\, e^{3x}$

(d) $y = c_1\, e^{3x} + c_2\, x\, e^{-3x}$

22. Find the general solution to $y'' + 4\, y' + 13\, y = 0$.

(a) $y = c_1\, e^{-2x}\, \sin(3x) + c_2\, e^{-2x}\cos(3x)$

(b) $y = c_1\, e^{-3x}\, \sin(2x) + c_2\, e^{-3x}\cos(2x)$

(c) $y = c_1\, e^{3x}\, \sin(2x) + c_2\, e^{3x}\cos(2x)$

(d) $y = c_1\, e^{2x}\, \sin(3x) + c_2\, e^{2x}\cos(3x)$

23. For the equation $y'' + 3\, y' + 2y = 5\, x + e^{-x} + 7\, \sin(4x)$, determine the form of the simplest particular solution if the method of undetermined coefficients is to be used.

(a) $y_p = A_1 + A_2\, x + A_3\, e^{-x} + A_4\, \cos(4x) + A_5\, \sin(4x)$

(b) $y_p = A_1 + A_2\, x + A_3\, e^{-x} + A_4\, x\, \cos(4x) + A_5\, x\, \sin(4x)$

(c) $y_p = A_1 + A_2\, x + A_3\, x\, e^{-x} + A_4\, \cos(4x) + A_5\, \sin(4x)$

(d) $y_p = A_1\, x + A_2\, x^2 + A_3\, x\, e^{-x} + A_4\, \cos(4x) + A_5\, \sin(4x)$

24. For the equation $y'' + 8\, y' + 16\, y = 7\, x^3\, e^{-4x}$, determine the form of the simplest particular solution if the method of undetermined coefficients is to be used.

(a) $y_p = A_1\, e^{-4x} + A_2\, x\, e^{-4x} + A_3\, x^2\, e^{-4x} + A_4\, x^3\, e^{-4x}$

(b) $y_p = A_1\, x\, e^{-4x} + A_2\, x^2\, e^{-4x} + A_3\, x^3\, e^{-4x} + A_4\, x^4\, e^{-4x}$

(c) $y_p = A_1\, x^2\, e^{-4x} + A_2\, x^3\, e^{-4x} + A_3\, x^4\, e^{-4x} + A_4\, x^5\, e^{-4x}$

(d) $y_p = A_1\, x^3\, e^{-4x} + A_2\, x^4\, e^{-4x} + A_3\, x^5 e^{-4x} + A_4\, x^6\, e^{-4x}$

25. Which of the following is *not* a particular solution to $y'' + 2y' = 5$?

(a) $3 + \frac{5}{2}x$ (b) $1 + \frac{5}{2}x$ (c) $\frac{5}{2} + e^{-2x}$ (d) $4 + \frac{5}{2}x + 3e^{-2x}$

26. Given $M = \begin{bmatrix} -2 & 1 & 3 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$, find the eigenvalues of M.

(a) $-3, -1, 2$ (b) $1, 2, 3$ (c) $-1, -2, 3$ (d) $-3, -2, 1$

27. Find the general solution to the ODE given by $y'''' - y = 0$.

(a) $y = c_1 e^x + c_2 x e^x + c_3 \cos x + c_4 \sin x$ (b) $y = c_1 e^{-x} + c_2 e^x + c_3 x e^x + c_4 x e^{-x}$

(c) $y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$ (d) $y = c_1 e^{-x} + c_2 e^x + c_3 x e^x + c_4 x^2 e^x$

28. If $e^A = \begin{bmatrix} \frac{1}{2} + \frac{e^4}{2} & -\frac{1}{4} + \frac{e^4}{4} \\ -1 + e^4 & \frac{1}{2} + \frac{e^4}{2} \end{bmatrix}$, determine e^{2A}.

(a) $\begin{bmatrix} 1 + e^4 & -\frac{1}{2} + \frac{e^4}{2} \\ -2 + 2e^4 & 1 + e^4 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{4} + \frac{e^4}{2} + \frac{e^8}{4} & \frac{1}{16} - \frac{e^4}{8} + \frac{e^8}{16} \\ 1 - 2e^4 + e^8 & \frac{1}{4} + \frac{e^4}{2} + \frac{e^8}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2} + \frac{e^{16}}{2} & -\frac{1}{4} + \frac{e^{16}}{4} \\ -1 + e^{16} & \frac{1}{2} + \frac{e^{16}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} + \frac{e^8}{2} & -\frac{1}{4} + \frac{e^8}{4} \\ -1 + e^8 & \frac{1}{2} + \frac{e^8}{2} \end{bmatrix}$

29. Find all linearly independent eigenvectors of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

30. Find the general solution to the ODE given by $[(D+1)^2 + 4]^3 y = 0$, where D denotes the differential operator d/dx.

(a) $y = \left(c_1 + c_2 x + c_3 x^2\right) e^{-x} \cos(2x) + \left(c_4 + c_5 x + c_6 x^2\right) e^{-x} \sin(2x)$

(b) $y = \left(c_1 + c_2 x + c_3 x^2\right) e^{x} \cos(2x) + \left(c_4 + c_5 x + c_6 x^2\right) e^{x} \sin(2x)$

(c) $y = \left(c_1 + c_2 x + c_3 x^2\right) e^{-3x} + \left(c_4 + c_5 x + c_6 x^2\right) e^{x}$

(d) $y = \left(c_1 + c_2 x + c_3 x^2\right) e^{3x} + \left(c_4 + c_5 x + c_6 x^2\right) e^{-x}$