Aktosun, Spring 2017, Math 3319, Exam 2 Answer Key

Questions 1-15: True/False. Questions 16-30: Multiple-Choice.

1. Let $\mathcal{M}_{m\times n}$ denote the vector space of $m \times n$ matrices over the field of real numbers. Then, $\mathcal{M}_{3\times 4}$ is a subspace of $\mathcal{M}_{3\times 5}$. F

2. Consider a set of 1403 vectors belonging to a vector space of dimension 1304. It is possible that such a set of ~~1304~~ 1403 vectors are linearly independent for some specific choice of those 1403 vectors. F

3. The three vectors $(2,3,5)$, $(-1,0,4)$, and $(0,3,13)$ are linearly dependent in $\mathbf{R}^3$. T

4. Let P_n denote the vector space of polynomials in x of degree n or less, with real-valued coefficients. The dimension of P_{302} is 302. F

5. The set of invertible 6×6 matrices with real-valued entries form a subspace of $\mathcal{M}_{6\times 6}$, the vector space of 6×6 matrices with real-valued entries. F

6. If A is a 7×9 matrix with real entries, then the null space of A must be a subspace of $\mathbf{R}^9$. T

7. The real-valued 13×13 matrices with zero trace form a subspace of $\mathcal{M}_{13\times 13}$, the vector space of 13×13 matrices with real-valued entries. T

8. Let A be a 7×7 matrix with complex-valued entries. We must always have the determinant of the transpose of A equal to the determinant of A. T

9. Let V be the vector space of 8×8 upper-triangular matrices with real-valued entries. Then, the dimension of V must be equal to 36. T

10. The set of vectors (x,y,z) in $\mathbf{R}^3$ satisfying $2x - y + z = 1$ must be a subspace of $\mathbf{R}^3$. F

11. Assume that a vector space V contains at least one nonzero vector. Then, V must contain infinitely many distinct vectors. T

12. Let A and B be two 89×89 matrices with the property that $AB = I$, where I denotes the 89×89 identity matrix. Then, we must always have $BA = I$. T

13. For any two $n \times n$ matrices A and B, we must always have $(A - B)^2 = A^2 - 2AB + B^2$. F

14. For any two diagonal matrices A and B having the same matrix sizes, we must always have $AB = BA$. T

15. Any 7×7 matrix with nonzero trace must be invertible. F

16. Assume that the matrix A has 3 columns and 5 rows and the matrix B has 5 columns and 2 rows. How many columns does the matrix BA have?

(a) 1 (b) 2 (c) 3 (d) 5

17. Find the reduced row-echelon form of the matrix $\begin{bmatrix} 1 & 2 & -1 & -2 \\ 3 & -3 & 33 & 12 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 7 & 2 \\ 0 & 1 & -4 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 7 & -2 \\ 0 & 1 & -4 & -2 \end{bmatrix}$

18. If A is the matrix given by $\begin{bmatrix} x & y \\ z & t \end{bmatrix}$, what is $-3\left(A^T\right)^2$, where T denotes the matrix transpose?

(a) $\begin{bmatrix} -3x & -3z \\ -3y & -3t \end{bmatrix}$ (b) $\begin{bmatrix} -3x^2 & -3y^2 \\ -3z^2 & -3t^2 \end{bmatrix}$ (c) $\begin{bmatrix} -3x^2 & -3z^2 \\ -3y^2 & -3t^2 \end{bmatrix}$

(d) $\begin{bmatrix} -3x^2 - 3yz & -3xz - 3zt \\ -3xy - 3yt & -3yz - 3t^2 \end{bmatrix}$

19. What is the value of the determinant of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -5 \\ -1 & 0 & 3 \end{bmatrix}$?

(a) 0 (b) 10 (c) 17 (d) 34

20. Consider the permutation tensor ϵ_{ijkl}, where the indices i, j, k, l run over all the integer values consisting of $1, 2, 3, 4$. What is the value of $5\,\epsilon_{1214} - \epsilon_{1342} + 7\,\epsilon_{3412} + 2\,\epsilon_{1432}$?

(a) 4 (b) 5 (c) 6 (d) 13

21. Which of the following is correct for the determinant of a 10×10 matrix A?

(a) $\det(-2A) = -2\,\det(A)$ (b) $\det(-2A) = 2\,\det(A)$

(c) $\det(-2A) = -2^{10}\,\det(A)$ (d) $\det(-2A) = 2^{10}\,\det(A)$

22. Let A, B, and C be three 4×4 matrices with determinants 2, -5, and 3, respectively. What is the value of the determinant of the matrix $\left((-B)^2 A^T C^3\right)^T$, where T denotes the matrix transpose?

(a) -30 (b) -1350 (c) 30 (d) 1350

23. Let A be the matrix equal to the matrix product $[1 \quad 2 \quad 3]^T\,[4 \quad 5 \quad 6]$, where T denotes the matrix transpose. What is the values of the determinant of A?

(a) 0 (b) 32 (c) 720 (d) does not exist

24. If the inverse of the matrix A is given by $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, then what is the solution x to the linear system $A\mathbf{x} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$?

(a) $\begin{bmatrix} -1/4 \\ -1/8 \\ -1/4 \end{bmatrix}$ (b) $\begin{bmatrix} -4 \\ -4 \\ -4 \end{bmatrix}$ (c) $\begin{bmatrix} -4 \\ -8 \\ -4 \end{bmatrix}$ (d) does not exists

25. Based on Cramer's rule, what is the value of y in the linear system given by

$$\begin{cases} ax + by = e \\ cx + dy = f. \end{cases}$$

(a) $\dfrac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}$ (b) $\dfrac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}$ (c) $\dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ (d) $\dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

26. If $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 7 \\ -4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, what is $-(2A - 3B)C$?

(a) $\begin{bmatrix} -27 & 17 \\ 10 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} 17 & -31 \\ 34 & -8 \end{bmatrix}$ (c) $\begin{bmatrix} 31 & -31 \\ 18 & -8 \end{bmatrix}$ (d) $\begin{bmatrix} -39 & 23 \\ -4 & -8 \end{bmatrix}$

27. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 3 \\ 2 & -1 & -5 \end{bmatrix}$. What is the value of $\left[2\det(A) - 3\operatorname{tr}(A) - 20\det(A^{-1})\right]^2$, where det and tr denote the determinant and trace, respectively?

(a) 36 (b) 81 (c) 225 (d) 324

28. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 1/2 & -1 \\ 0 & 1/2 & 1/4 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1/2 & -1 & 1/2 \\ 0 & -1 & -2 \\ 0 & 0 & 1/2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 & -5 \\ 0 & 1/2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

(d) does not exist

29. Find a basis for P_2, the vector space of polynomials in x of degree 2 or less, with real-valued coefficients.

(a) $\{1 + x + x^2, 1 - x + x^2, 2x\}$ (b) $\{1 + x + x^2, 1 - x + x^2, 1\}$

(c) $\{x + x^2, x - x^2, x\}$ (d) $\{1 + x^2, 1 - x^2, 7\}$

30. Find a basis for $\mathcal{M}_{2\times 2}$, the vector space of 2×2 matrices with real-valued entries.

(a) $\left\{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(b) $\left\{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(d) $\left\{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$