Aktosun, Spring 2017, Math 3319, Exam 1 Answer Key

Questions 1-15: True/False. **Questions 16-30: Multiple-Choice.**

1. The function $y = 0$ is always a solution to a linear homogeneous ODE. T

2. The ODE $\dfrac{dy}{dx} = (x + y + 5)^2$ is separable. F

3. If $y(x)$ is a solution to an n-th order ODE and contains n arbitrary constants, then it must be the general solution to that ODE. T

4. A first-order ODE can never be linear, separable, and exact at the same time. F

5. The ODE $y' = e^{5x-7y}$ is separable. T

6. The ODE $M(x, y)\, dx + N(x, y)\, dy = 0$ is exact if and only if $\dfrac{\partial M(x, y)}{\partial x} = \dfrac{\partial N(x, y)}{\partial y}$. F

7. For any linear ODE, the sum of any two solutions is also a solution. F

8. The only particular solution to a second-order linear homogeneous ODE is the zero solution. F

9. The ODE $x^8 \dfrac{d^8 y}{dx^8} - 3y = 0$ is a Cauchy-Euler equation and has one solution of the form $y = x^r$ for some constant r. T

10. The function e^{x^2} is an integrating factor for the ODE $y' - 2xy = x^7$. F

11. $(yz)' = y'z + yz'$ is a differential identity rather than a differential equation. T

12. The ODE $\dfrac{dy}{dx} = \dfrac{x^8 y - 23x^2 y^7}{x^4 y^5 + 3y^9 - 8x^9}$ is a first-order homogeneous equation. T

13. The ODE $\dfrac{dy}{dx} = \sqrt{x}\, y + 5\sqrt{xy}$ is a Bernoulli differential equation. T

14. The ODE $\frac{dy}{dx} = \frac{4xy + 5y^2}{xy + 3x^2}$ can be transformed into a separable ODE in x and z by using the substitution $y = xz$.

T

15. The electric charge $Q(t)$ on one side of a capacitor plate in an LRC circuit with the voltage source $V(t)$ is described by the ODE $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V(t)$.

T

16. The general solution to $y' = xy^2$ has the form

(a) $y = \pm\sqrt{e^{x^2/2+c}}$ (b) $y = \frac{x^2y^2}{2} + c$ (c) $y = \frac{-2}{x^2+c}$. (d) $y = \frac{x^2y^3}{6} + c$

17. Which of the following is a linear ODE?

(a) $y\frac{d^2y}{dx^2} = 1$ (b) $\frac{d^2y}{dx^2} - y = 1$ (c) $y\frac{d^2y}{dx^2} - \frac{1}{y} = y$ (d) $\frac{d^2y}{dx^2} = \sin y$.

18. The general solution to $y' + y = e^{2x}$ has the form

(a) $y = ce^{-x} + \frac{e^{2x}}{3}$ (b) $y = \frac{e^{2x} + c}{2(1+x)}$ (c) $y = \left(c + \frac{1}{3}\right)e^{2x}$ (d) $y = c\,e^{2x}$

19. The general solution to $y' = y^2$ has the form

(a) $y = \frac{c}{x+1}$ (b) $y = \frac{1}{x+c}$ (c) $y = \frac{-1}{x+c}$. (d) $y = \frac{c}{x-1}$

20. An integrating factor for $y' + 8x^3y = \sin x$ is given by

(a) $2x^4$ (b) $\ln(x^4)$ (c) x^4 (d) $5e^{2x^4}$

21. Which of the following is an exact ODE?

(a) $(x+y)\,dx - dy = 0$ (b) $3y\,dx - 3x\,dy = 0$ (c) $2xy\,dx + (3+x^2)\,dy = 0$

(d) $y^2\,dx + (2x - 2y)\,dy = 0$

22. The general solution to $\frac{dy}{dx} = e^{x-y}$ has the form

(a) $y = \ln(x+c)$ (b) $y = \ln(-e^x+c)$ (c) $y = \ln(e^{-x}+c)$ (d) $y = \ln(e^x+c)$

23. The equation $y^2 = cx$ constitutes an implicit general solution to the ODE given by

(a) $xy' = 2y$ (b) $yy' = 2x$ (c) $2xy' = y$ (d) $2yy' = x$

24. The general solution to $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$ has the form

(a) $y^2 = \dfrac{x^2}{4} + \dfrac{c}{x}$ (b) $y = \dfrac{x^3}{4} + \dfrac{c}{x}$ (c) $y = \dfrac{x^2}{4} + \dfrac{c}{x^2}$ (d) $y = \dfrac{x^3}{4} + c$

25. The general solution to $\dfrac{dy}{dx} = \dfrac{3}{\sqrt{2x+7}}$ has the form

(a) $y = 3\sqrt{2x+7} + c$ (b) $y = \dfrac{3}{\sqrt{2x+7}} + c$ (c) $y = \sqrt{6x+14} + c$

(d) $y = \dfrac{1}{\sqrt{6x+14}} + c$

26. The general solution to $\dfrac{dy}{dx} + 2xy = 2x$ has the form

(a) $y = 1 + ce^{x^2}$ (b) $y = 1 + ce^{-x^2}$ (c) $y = (1+c)e^{-x^2}$ (d) $y = (1+c)e^{x^2}$

27. The general solution to $x^2\dfrac{dy}{dx} - 2xy = 3$ has the form

(a) $y = \dfrac{3}{2} + \dfrac{c}{x^2}$ (b) $y = -\dfrac{3}{2} + \dfrac{c}{x^2}$ (c) $y = \dfrac{1}{x} + cx^2$ (d) $y = -\dfrac{1}{x} + cx^2$

28. The solution to $\dfrac{dy}{dx} = 3x^2 + 9$ satisfying $y(1) = 2$ is given by

(a) $y = x^3 + 9x - 8$ (b) $y = x^2 + 9 - 8x$ (c) $y = x^2 + 9x - 8$

(d) $y = x^3 + 9 - 8x$

29. The general solution to $3x\dfrac{dy}{dx} + y = 12x$ is given by

(a) $y = 3x + c$ (b) $y = 3x + cx^{1/3}$ (c) $y = 3x + cx^{-1/3}$ (d) $y = \dfrac{3}{x} + c$

30. The general solution to $y^4\,dx + 4xy^3\,dy = 0$ is given by

(a) $y = cx^{-1/4}$ (b) $y = cx^{1/4}$ (c) $y = x^{-1/4} + c$ (d) $y = x^{1/4} + c$